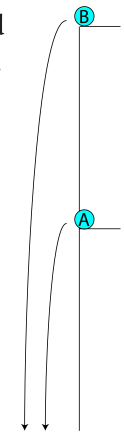
AP1 WEP

1. Bowling Ball A is dropped from a point halfway up a cliff. A second identical bowling ball, B, is dropped simultaneously from the top of the cliff. Comparing the bowling balls at the instant they reach the ground, which of the following are correct? Neglect air resistance.



(A) Ball A has half the kinetic energy and takes half the time to hit the ground as Ball B.
(B) Ball A has half the kinetic energy and takes one-fourth the time to hit the ground as Ball B.
(C) Ball A has half the final velocity and takes half the time to hit the ground as Ball B.
(D) Ball A has one-fourth the final velocity and takes one-fourth the time to hit the ground as Ball B.
(E) None of these are correct.

Answer: E

The final velocities of the balls are given by $v = \sqrt{2gh}$.

The final velocity of B is related to the square root of the height, therefore the final velocity of B is $\sqrt{2}$ times the final velocity of A, eliminating choices C and D. Further, the kinetic energy of Ball A is half the kinetic energy of Ball B at the instant the balls reach the ground. The time it takes for the balls to reach the ground is also related to the square root of the height, therefore the time for B to hit the ground is $\sqrt{2}$ times the time for A to hit the ground, eliminating choices A and B. E must be the correct answer.

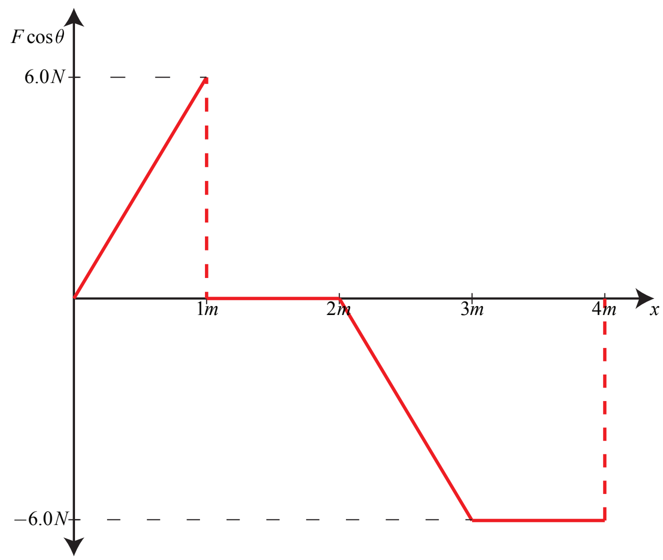
EK: 5.B.4 The internal energy of a system includes the kinetic energy of the objects that make up the system and the potential energy of the configuration of the objects that make up the system.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 5.B.4.2 The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system.

AP1 WEP

2. A net force (Fcosθ) acts on an object in the x-direction while moving over a distance of 4 meters along the axis, depicted in the graph at right.



(a) Find the work done by the force in the interval from 0.0 to 1.0 m.

(b) Find the work done by the force in the interval from 1.0 to 2.0 m.

(c) Find the work done by the force in the interval from 2.0 to 4.0 m.

(d) At what position(s) is the object moving with the largest speed? Explain your answer.

Answers:

(a) 3 J (b) 0 J (c) -9 J

(d) The object is moving with the largest speed at the positions of 1 to 2 m. From 0 to 1 m the force is increasing the speed of the object where it reaches a maximum value. From 1 to 2 m there is no force acting on the object therefore it is moving at a constant speed. From 2 to 4 m the negative force is slowing the object down since the object is still moving in the positive x direction.

EK: 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work.

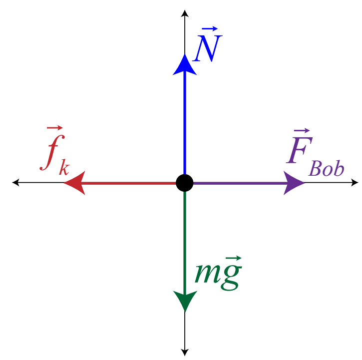
SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 4.C.2.1 The student is able to make predictions about the changes in the mechanical energy of a system when a component of an external force acts parallel or antiparallel to the direction of the displacement of the center of mass.

3. Bob pushes a box across a horizontal surface at a constant speed of 1 m/s. If the box has a mass of 30 kg, find the power Bob supplies given the coefficient of kinetic friction is 0.3.

Answer: 90 W

First, draw a free body diagram for the situation.



Next, recognizing the box is moving at constant speed and is therefore experiencing no net force, apply Newton's 2nd Law in the x- and y-directions to solve for Bob's applied force.

$$F_{net_X} = F_{Bob} - f_k = ma_x \xrightarrow{a_x=0} F_{Bob} = f_k = \mu_k N$$

$$F_{net_Y} = N - mg = 0 \rightarrow N = mg$$

$$F_{Bob} = \mu_k N = \mu_k mg = (0.3)(30kg)(10\,{}^m\!/\!{}_{s^2}) = 90N$$

Finally, use the relationship between power, force, and velocity to determine the power supplied.

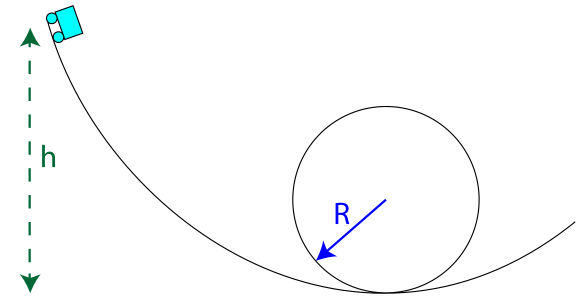
$$P = \frac{\Delta E}{\Delta t} = \frac{Fd\cos\theta}{\Delta t} \xrightarrow[\cos\theta=1]{\bar{v}=\frac{d}{\Delta t}} P = F\bar{v} = (90N)(1\,{}^m\!/\!{}_s) = 90W$$

EK: 3.B.2 Free-body diagrams are useful tools for visualizing forces being exerted on a single object and writing the equations that represent a physical situation. 5.B.5 Energy can be transferred by an external force exerted on an object or system that moves the object or system through a distance; this energy transfer is called work. Energy transfer in mechanical or electrical systems may occur at different rates. Power is defined as the rate of energy transfer into, out of, or within a system.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 3.B.2.1 The student is able to create and use free-body diagrams to analyze physical situations to solve problems with motion qualitatively and quantitatively. 5.B.5.5 The student is able to predict and calculate the energy transfer to (i.e., the work done on) an object or system from information about a force exerted on the object or system through a distance.

4. A roller coaster car begins at rest at height h above the ground and completes a loop along its path. In order for the car to remain on the track throughout the loop, what is the minimum value for h in terms of the radius of the loop, R? Assume frictionless.



Answer: h must be greater than or equal to 5R/2.

First, use the law of conservation of energy to find the speed of the cart at the top of the loop.

$$U_i = K_f + U_f \rightarrow mgh = \tfrac{1}{2}mv_f^2 + mg(2R) \rightarrow 2gh = v_f^2 + 4gR \rightarrow v_f^2 = 2g(h-2R)$$

Next, recognize for the cart to stay on the loop and not fall off, the centripetal acceleration must be equal to or greater than the acceleration due to gravity.

$$\frac{v^2}{R} \geq g \rightarrow v^2 \geq gR \rightarrow 2g(h-2R) \geq gR \rightarrow (h-2R) \geq \frac{R}{2} \rightarrow h \geq \frac{5R}{2}$$

EK: 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy. 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 7.2 The student can connect concepts in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

LO: 4.C.1.1 The student is able to calculate the total energy of a system and justify the mathematical routines used in the calculation of component types of energy within the system whose sum is the total energy. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

5. If you throw a rock straight up outside, it eventually returns to your hand with the same speed that it had when it left, neglecting air resistance. What would happen if you were to throw the rock straight up in the same way, but while inside the classroom? Compared to the speed with which it left your hand, after rebounding off of the ceiling it would return to your hand with...

(A) a higher speed.
(B) a lower speed.
(C) the same speed, just like when you are outside.
(D) more information needed

Explain your reasoning.

Answer: (B) a lower speed. Analyzing this problem through the lens of conservation of energy, some energy is lost in the collision to sound, internal energy, work done on the ceiling, etc., so when the rock returns to your hand, it has to have less energy than it began with.

EK: 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy. 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.C.1.2 The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

6. Is it possible to give a block a push and then have it slide up an incline at a constant speed (without you continuing to push on it)? Explain your reasoning.

(A) No.
(B) Yes, but only if there is friction between the block and the incline.
(C) Yes, but only if there is no friction between the block and the incline.

Explain your reasoning.

Answer: (A) No. The block has a set amount of kinetic energy at the bottom of the incline. Once the initial push ends, no more work is done on the block to increase its kinetic energy. As the block travels higher up the ramp, KE is converted to gravitational potential energy, so the block must slow down.

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7. Is it possible to give a block a push and then have it slide down an incline at a constant speed (without continuing to push on it)?

(A) No.
(B) Yes, but only if there is friction between the block and the incline.
(C) Yes, but only if there is no friction between the block and the incline.

Explain your reasoning.

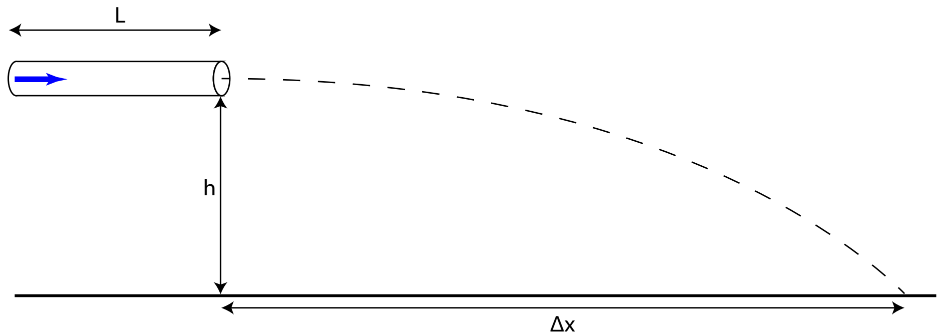
Answer: (B) Yes, but only if there is friction between the block and the incline. The block has gravitational potential energy and kinetic energy at the top of the incline. As it travels down the incline, gravitational potential energy is converted into internal energy through friction. If this were a frictionless incline, the block would accelerate as the gravitational potential energy became kinetic energy.

EK: 4.C.1 The energy of a system includes its kinetic energy, potential energy, and microscopic internal energy. Examples should include gravitational potential energy, elastic potential energy, and kinetic energy. 5.A.2 For all systems under all circumstances, energy, charge, linear momentum, and angular momentum are conserved. For an isolated or a closed system, conserved quantities are constant. An open system is one that exchanges any conserved quantity with its surroundings.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.C.1.2 The student is able to predict changes in the total energy of a system due to changes in position and speed of objects or frictional interactions within the system. 5.A.2.1 The student is able to define open and closed systems for everyday situations and apply conservation concepts for energy, charge, and linear momentum to those situations.

8. A dart of mass m is accelerated horizontally through a tube of length L situated a height h above the ground by a constant force F. Upon exiting the tube, the dart travels a horizontal distance Δx before striking the ground, as depicted in the diagram below.



(a) Develop an expression for the velocity of the dart, v, as it leaves the tube in terms of Δx, h, and any fundamental constants.

(b) Derive an expression for the kinetic energy of the dart as it leaves the tube in terms of m, Δx, h, and any fundamental constants.

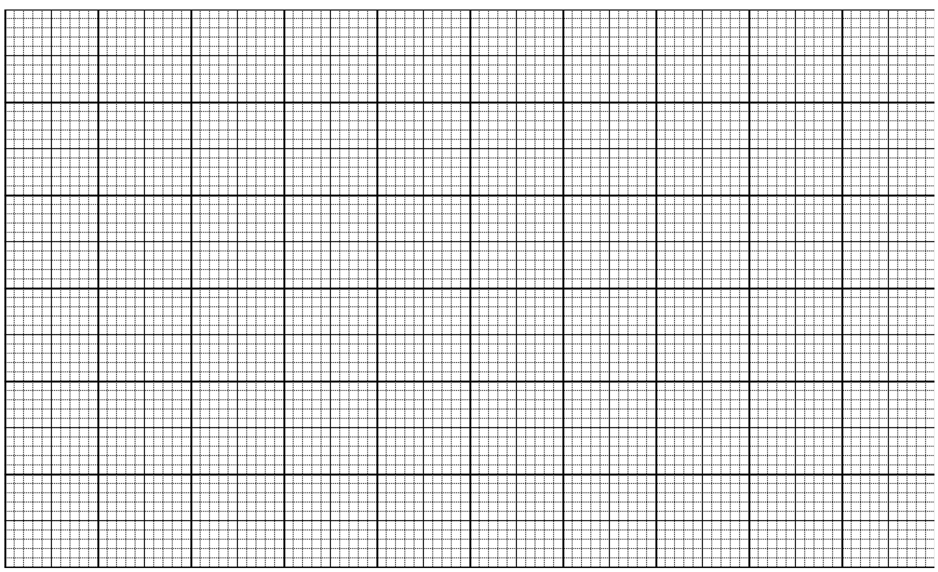
(c) Derive an expression for the work done on the dart in the tube in terms of F and L.

(d) Derive an expression for the height of the tube above the ground in terms of m, Δx, L, F, and any fundamental constants.

An experiment is then performed in which the length of the tube, *L*, is varied, resulting in the dart traveling various horizontal distances Δx which are recorded in the table below.

Trial	1	2	3	4	5	6
Tube Length *L* (m)	0.2	0.5	0.8	1.2	1.7	2.0
Horizontal Distance Δx (m)	3.5	5.5	7.0	8.6	10.2	11.1

(e) Use the grid below to plot a linear graph of Δx^2 as a function of *L*. Use the empty boxes in the data table, as appropriate, to record the calculated values you are graphing. Label the axes as appropriate, and place numbers on both axes.



(f) From the graph, obtain the height of the tube given the mass of the dart is 20 grams and the constant force applied in the tube is 2 newtons.

Answer:

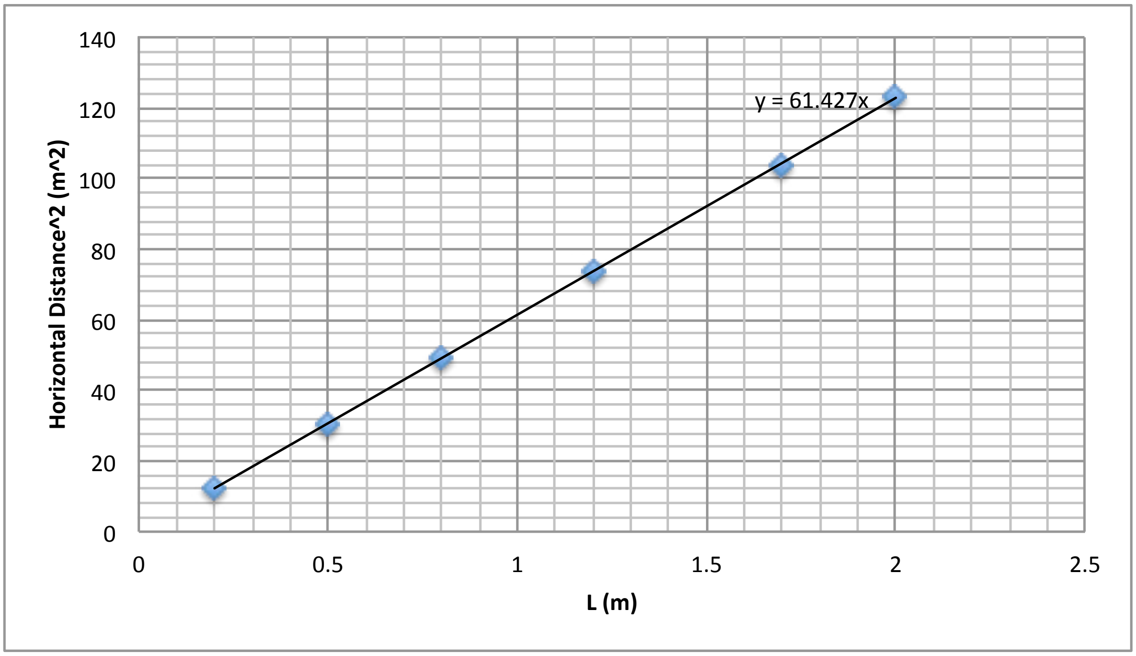
(a) $v = \frac{\Delta x}{\sqrt{2h/g}}$

(b) $K = \frac{mg\Delta x^2}{4h}$

(c) $W = FL$

(d) $h = \frac{mg\Delta x^2}{4FL}$

(e)



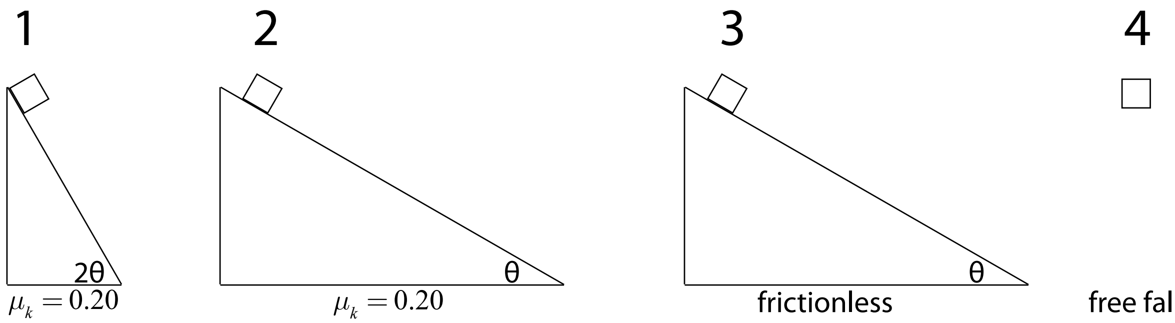
(f) $h = \frac{\Delta x^2}{L}\frac{mg}{4F} = slope\left(\frac{mg}{4F}\right) = 61\left(\frac{(0.02)(9.8)}{4(2)}\right) = 1.5m$

EK: 3.E.1 The change in the kinetic energy of an object depends on the force exerted on the object and on the displacement of the object during the interval that the force is exerted. 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work. 5.B.5 Energy can be transferred by an external force exerted on an object or system that moves the object or system through a distance; this energy transfer is called work.

SP: 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 4.4 The student can evaluate sources of data to answer a particular scientific question. 5.3 The student can evaluate the evidence provided by data sets in relation to a particular scientific question.

LO: 3.E.1.4 The student is able to apply mathematical routines to determine the change in kinetic energy of an object given the forces on the object and the displacement of the object. 4.C.2.2 The student is able to apply the concepts of Conservation of Energy and the Work-Energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system. 5.B.5.1 The student is able to design an experiment and analyze data to examine how a force exerted on an object or system does work on the object or system as it moves through a distance.

9. Identical metal blocks initially at rest are released in various environments as shown in scenarios A through D below. In all cases, the blocks are released from a height of 2m. Neglect air resistance.



Rank the scenarios from least kinetic energy to greatest kinetic energy at the instant before the block reaches the ground.

(A) 2 < 3 < 1 < 4
(B) 2 < 1 < 3 = 4
(C) 2 < 1 < 3 < 4
(D) 1 = 2 < 3 = 4

Answer: (B) 2 < 1 < 3 = 4

Scenarios 3 and 4 both start at the same height, with the same gravitational potential energy, and without friction, they must have the same mechanical energy at height 0, therefore they have the same kinetic energy. Scenarios 1 and 2 have both lost some energy to friction, so they will have a smaller amount of kinetic energy at the bottom. At a greater (steeper) angle, scenario 1 has a smaller frictional force over a shorter distance, therefore less work is done by friction, leaving more kinetic energy at the bottom.

EK: 4.C.2 Mechanical energy (the sum of kinetic and potential energy) is transferred into or out of a system when an external force is exerted on a system such that a component of the force is parallel to its displacement. The process through which the energy is transferred is called work.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.C.2.2 The student is able to apply the concepts of Conservation of Energy and the Work-Energy theorem to determine qualitatively and/or quantitatively that work done on a two-object system in linear motion will change the kinetic energy of the center of mass of the system, the potential energy of the systems, and/or the internal energy of the system.