22) A solid uniform cylinder is rolling without slipping. What fraction of its kinetic energy is rotational?

A) 1/3
B) 2/3
C) 1/2
D) 1/4
E) 3/4

$$K_{total} = K_{translation} + K_{rotational}$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}(\tfrac{1}{2}mr^2)(\tfrac{v^2}{r^2})$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{4}mv^2$$
$$= \tfrac{3}{4}mv^2$$

$$\frac{K_{rotational}}{K_{total}} = \frac{1/4}{3/4} = \frac{1}{3}$$

23) A hoop is rolling without slipping along a horizontal surface with a forward speed of 5.50 m/s when it starts up a ramp that makes an angle of 25.0° with the horizontal. What is the speed of the hoop after it has rolled 3.00 m up as measured along the surface of the ramp?

A) 4.22 m/s
B) 1.91 m/s
C) 2.06 m/s
D) 3.79 m/s
E) 8.02 m/s

$$K_{rot} + K_{tran} = K_{rot} + K_{tran} + U$$
$$mv_i^2 = mv_f^2 + mgh$$
$$v_f^2 = v^2 - gh$$
$$= (5.5)^2 - (9.8)(3\sin 25°)$$
$$v = \sqrt{} = 4.22 \text{ m/s}$$
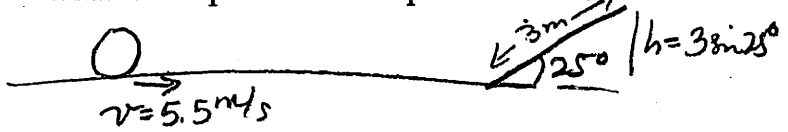
$$K_{total} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}(mr^2)(\tfrac{v^2}{r^2})$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2$$
$$= mv^2$$

$v = 5.5$ m/s   3 m   25°   h = 3 sin 25°

24) A solid uniform sphere is rolling without slipping along a horizontal surface with a speed of 5.5 m/s when it starts up a ramp that makes an angle of 25° with the horizontal. What is the speed of the sphere after it has rolled 3.0 m up as measured along the surface of the ramp?

A) 4.0 m/s
B) 8.0 m/s
C) 1.9 m/s
D) 2.2 m/s
E) 3.5 m/s

$$K_{total} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(\tfrac{2}{5}mv^2)$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{5}mv^2$$
$$= \tfrac{7}{10}mv^2$$

$$\tfrac{7}{10}mv_i^2 = \tfrac{7}{10}mv_f^2 + mgh$$
$$v_i^2 = v_f^2 + \tfrac{10}{7}gh$$
$$v_f^2 = v_i^2 - \tfrac{10}{7}gh$$
$$= 12.51$$
$$v_f = \sqrt{} = 3.5 \text{ m/sec}$$

25) A uniform solid disk is released from rest and rolls without slipping down an inclined plane that makes an angle of 25° with the horizontal. What is the forward speed of the disk after it has rolled 3.0 m, measured along the plane?

the plane?
A) 2.0 m/s
B) 3.5 m/s
C) 4.1 m/s
D) 5.7 m/s
E) 6.3 m/s

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}(\tfrac{1}{2}mr^2)(\tfrac{v^2}{r^2})$$
$$gh = \tfrac{1}{2}v^2 + \tfrac{1}{4}v^2$$
$$gh = \tfrac{3}{4}v^2$$
$$v = \sqrt{\tfrac{4}{3}gh} = \sqrt{\tfrac{4}{3}(9.8)(3\sin 25°)}$$
$$= 4.1 \text{ m/sec}$$

1)   A,C 2) B 3) A 4) C 5) A 6) A 7) 3.49 rad/s 8) B 9) a) 0.627 rad/s b) 1.18 m/s² 10) B 11) A 12) B 13) A
14) 24.9 kg*m² 15) A 16) A 17) D 18) 1.6 m/s² 19) a) 1100N b) 710 N 20) A 21) 9 rad/s b) 0.089 J c) 0.020 kg*m² 22) A 23) A 24) E 25) C

use eqns. derived in #16

17) A cinder block of mass $m = 4.0$ kg is hung from a nylon string that is wrapped around a frictionless pulley having the shape of a cylindrical shell, as shown in the figure. If the cinder block accelerates downward at 4.90 m/s² when it is released, what is the mass M of the pulley?

$I = \frac{1}{2}MR^2$

T

m $4kg$ $\downarrow a = 4.9 m/s^2$

A) 2.0 kg
B) 4.0 kg
C) 6.0 kg
D) 8.0 kg
E) 10 kg
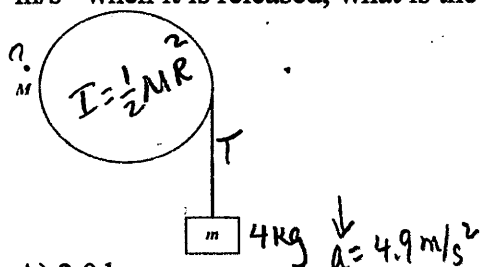
$T = mg - ma = 4(9.8) - 4(4.9) = 19.6 N$

$\frac{1}{2}Ma = T \Rightarrow M = \frac{2T}{a}$

$= \frac{2(19.6 N)}{4.9 m/s^2}$

$= 8 kg$

use eqns derived in #16

18) A uniform, solid, 100-kg cylinder with a diameter of 1.0 m is mounted so it is free to rotate about fixed, horizontal, frictionless axis that passes through the centers of its circular ends. A 10-kg block is hung from a very light thin cord wrapped around the cylinder's circumference. When the block is released, the cord unwinds and the block accelerates downward, as shown in the figure. What is the acceleration of the block? $1.6 m/s^2$

$m = 100 kg$

$m = 10 kg$
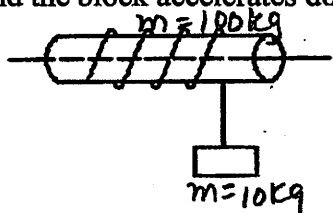
$a = \frac{2mg}{2m+M} = \frac{2(10 kg)(9.8 m/s^2)}{2(10 kg) + 100 kg}$

$= 1.63 m/s^2$

$\uparrow F_1$ $\qquad$ $\uparrow F_2$
$pivot$
$27 kg$ $82 kg$
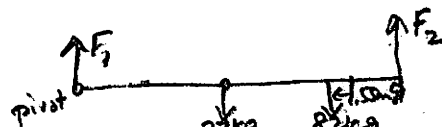
19) An 82-kg painter stands on a long horizontal board 1.55 m from one end. This 27-kg board is uniform, 5.5 m long, and supported at each end by vertical posts.
(a) What is the magnitude of the *total* force provided by *both* posts? $1100 N$
(b) With what force does the post that is closest to the painter push upward on the board? $F_2(5.5)$

$\Sigma F_y = 0$ $\quad F_1 + F_2 = 27(9.8) + 82(9.8)$ $\quad \Sigma \tau_t = \tau_{cc}$ $\quad 27(9.8)(\frac{5.5}{2}) + 82(9.8)(3.95) = F_2(5.5)$
$= 1068.2 N$ $\qquad F_2 = 709 N$

20) Consider a solid uniform sphere of radius R and mass M rolling without slipping. Which form of its kinetic energy is larger, translational or rotational?
A) Translational kinetic energy is larger.
B) Rotational kinetic energy is larger.
C) Both are equal.
D) You need to know the speed of the sphere to tell.

$K_{translational} = \frac{1}{2}mv^2$
$K_{rotational} = \frac{1}{2}I\omega^2$

21) A uniform ball with diameter of 10 cm rolls without slipping on a horizontal tabletop. The moment of inertia of the ball about an axis through its center is 2.2×10^{-3} kg · m², and the translational speed of its center is 0.45 m/s.
(a) What is its angular speed of the ball about its center of mass? $9 rad/s$
(b) What is the rotational kinetic energy of the ball? $0.089 J$
(c) What is the ball's angular momentum about its center of mass? $0.020 kg \cdot m^2$

(a) $\omega = \frac{v}{R} = \frac{0.45}{0.05}$ (b) $K_{rot} = \frac{1}{2}I\omega^2$
$= \frac{1}{2}(2.2 \times 10^{-3})(9)^2$

(c) $L = I\omega$
$= (2.2 \times 10^{-3})(9)$

12) A rolling wheel of diameter of 68 cm slows down uniformly from 8.4 m/s to rest over a distance of 115 m. What is the magnitude of its angular acceleration if there was no slipping?

A) 1.8 rad/s²
B) 0.90 rad/s²
C) 5.7 rad/s²
D) 11 rad/s²

$\alpha = \dfrac{a}{R}$

$R = 0.34 \text{ cm}$

$v_f^2 = v_0^2 + 2ad$

$a = \dfrac{v_0^2}{2d} = \dfrac{(8.4\,m/s)^2}{2(115\,m)} =$

13) A machinist turns on the power to a grinding wheel at time $t = 0$ s. The wheel accelerates uniformly from rest for 10 s and reaches the operating angular speed of 38 rad/s. The wheel is run at that angular speed for 30 s and then power is shut off. The wheel slows down uniformly at 2.1 rad/s² until the wheel stops. In this situation, what is the angular acceleration of the wheel between $t = 0$ s and $t = 10$ s?

A) 3.8 rad/s²
B) 4.6 rad/s²
C) 5.3 rad/s²
D) 6.1 rad/s²
E) 6.8 rad/s²

$\alpha = \dfrac{\Delta\omega}{t} = \dfrac{38}{10} = 3.8 \text{ rad}/s^2$

14) In the figure, a weightlifter's barbell consists of two identical small but dense spherical weights, each of mass 50 kg. These weights are connected by a thin 0.96-m rod with a mass of 24 kg. Find the moment of inertia of the barbell through the axis perpendicular to the rod at its center, assuming the two weights are small enough to be treated as point masses. 24.9 kg·m²

|←—0.96 m—→|
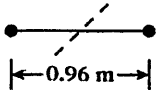
$I = m_1 r_1^2 + m_2 r_2^2 + \dfrac{1}{12} ML^2$

$= 50(.48)^2 + 50(.48)^2 + \dfrac{1}{12}(24)(0.96)^2$

$= 24.9 \text{ kg}\cdot m^2$

15) A potter's wheel has the shape of a solid uniform disk of mass 7.0 kg and radius 0.65 m. It spins about an axis perpendicular to the disk at its center. A small 2.1 kg lump of very dense clay is dropped onto the wheel at a distance 0.41 m from the axis. What is the moment of inertia of the system about the axis of spin?
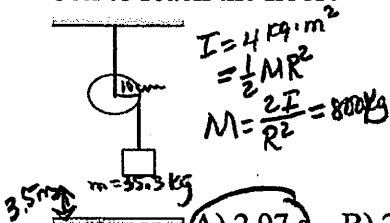
A) 1.8 kg · m²
B) 1.5 kg · m²
C) 0.40 kg · m²
D) 2.5 kg · m²

disk $I = \dfrac{1}{2}MR^2 = 1.48$

clay $I = Mr^2 = 0.35$

$I_{total} = 1.8 \text{ kg}\cdot m^2$

16) As shown in the figure, a 35.30-kg box is attached to a light string that is wrapped around a cylindrical frictionless spool of radius 10.0 cm and moment of inertia 4.00 kg · m². The spool is suspended from the ceiling, and the box is then released from rest a distance 3.50 m above the floor. How long does it take for the box to reach the floor?

A) 2.97 s
B) 2.85 s
C) 0.892 s
D) 4.18 s
E) 5.89 s

$mg - T = ma$
$T = mg - ma$

$I = 4 \text{ kg}\cdot m^2$
$= \dfrac{1}{2}MR^2$
$M = \dfrac{2I}{R^2} = 800\,kg$

$I\alpha = TR$
$\dfrac{1}{2}MR^2\left(\dfrac{a}{R}\right) = TR$
$\dfrac{1}{2}Ma = T$

$\dfrac{1}{2}Ma = mg - ma$
$Ma = 2mg - 2ma$
$Ma + 2ma = 2mg$
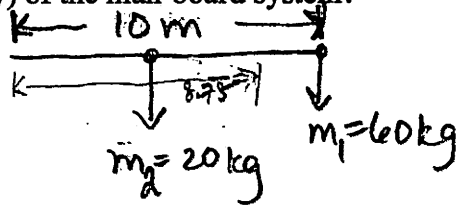$a = \dfrac{2mg}{2m + M} = \dfrac{2(35.3)(9.8)}{2(35.3) + 800} = 0.795\,m/s^2$

$\Delta y = v_0 t + \dfrac{1}{2}at^2 \Rightarrow t = \sqrt{\dfrac{2y}{a}} = \sqrt{\dfrac{2(3.5)}{0.795}}$

$= 2.97 \text{ sec}$

5) A 60.0-kg man stands at one end of a 20.0-kg uniform 10.0-m long board. How far from the man is the center of mass (or center of gravity) of the man-board system?

(A) 1.25 m
B) 2.50 m
C) 5.00 m
D) 7.50 m
E) 9.00 m

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{(m_1 + m_2)}$$
$$= \frac{60(10) + 20(5)}{60 + 20}$$
$$\boxed{\bar{x} = 8.75}$$

(diagram of board, 10 m, with $m_2 = 20\,kg$ and $m_1 = 60\,kg$, 8.75)

6) Through how many degrees does a 33 rpm turntable rotate in 0.32 s?

(A) 63°
B) 35°
C) 46°
D) 74°

$$\Delta\theta = \omega t = \left(33\,\frac{rev}{min}\right)\left(\frac{1\,min}{60\,sec}\right)\left(\frac{360°}{1\,rev}\right)(0.32\,sec)$$
$$= 63°$$

7) Express the angular speed of an old 33 1/3 rpm LP in rad/s. $3.49\ rad/s$

$$33.3\,\frac{rev}{min}\left|\frac{2\pi\,rad}{1\,rev}\right|\frac{1\,min}{60\,sec}$$

8) When a fan is turned off, its angular speed decreases from 10 rad/s to 6.3 rad/s in 5.0 s. What is the magnitude of the average angular acceleration of the fan?

A) 0.86 rad/s²
(B) 0.74 rad/s²
C) 0.37 rad/s²
D) 11 rad/s²
E) 1.2 rad/s²

$$\alpha = \frac{\Delta\omega}{t} = \frac{\left|6.3\,\frac{rad}{sec} - 10\,\frac{rad}{sec}\right|}{5\,sec}$$
$$= 0.74\ rad/s^2$$

9) When Mary is 3.00 m from the center of a merry-go-round, her tangential speed is a constant 1.88 m/s.
(a) What is her angular speed in rad/s? $0.627\ rad/s$ $\omega = V/R$
(b) What is the magnitude of her ~~linear~~ centripetal acceleration? $1.18\ m/s^2$ $a = \frac{v^2}{r}$

10) A child is riding a merry-go-round that is turning at 7.18 rpm. If the child is standing 4.65 m from the center of the merry-go-round, how fast is the child moving?

A) 5.64 m/s
(B) 3.50 m/s
C) 0.556 m/s
D) 1.75 m/s
E) 1.80 m/s

$$V = \omega R = \left(7.18\,\frac{rev}{min}\right)\left|\frac{1\,min}{60\,sec}\right|\frac{2\pi\,rad}{rev}(4.65\,m)$$

11) A string is wound tightly around a fixed pulley having a radius of 5.0 cm. As the string is pulled, the pulley rotates without any slipping of the string. What is the angular speed of the pulley when the string is moving at 5.0 m/s?

(A) 100 rad/s
B) 50 rad/s
C) 25 rad/s
D) 20 rad/s
E) 10 rad/s

$$\omega = \frac{V}{R} = \frac{5\,m/s}{0.05\,m}$$

NAME **Key**   DATE_____

1) When a rigid object rotates about a fixed axis, what is true about all the points in the object? (There could be more than one correct choice.)

A) They all have the same angular speed.
B) They all have the same tangential speed.
C) They all have the same angular acceleration.
D) They all have the same tangential acceleration.
E) They all have the same radial acceleration.

2) The figure shows scale drawings of four objects, each of the same mass and uniform thickness, with the mass distributed uniformly. Which one has the greatest moment of inertia when rotated about an axis perpendicular to the plane of the drawing at point P?

largest r

A B C D
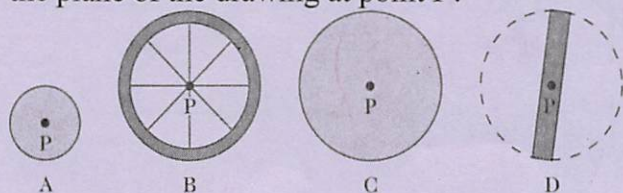
A) A
B) B
C) C
D) D
E) The moment of inertia is the same for all of these objects.

3) Two forces produce equal torques on a door about the door hinge. The first force is applied at the midpoint of the door; the second force is applied at the doorknob. Both forces are applied perpendicular to the door. Which force has a greater magnitude?

A) the first force (at the midpoint)
B) the second force (at the doorknob)
C) The two forces are equal.

$\tau_1 = \tau_2$; $r_1 < r_2$ $\therefore F_1 > F_2$

4) Five forces act on a rod that is free to pivot at point P, as shown in the figure. Which of these forces is producing a counter-clockwise torque about point P? (There could be more than one correct choice.)
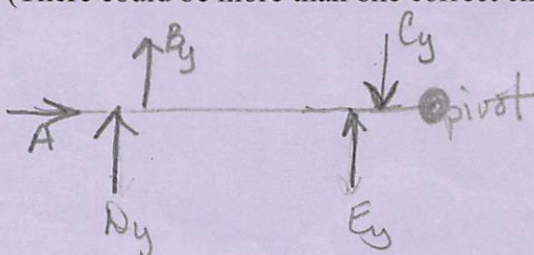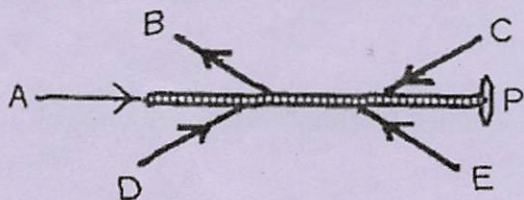
A) force A
B) force B
C) force C
D) force D
E) force E